

# ON 3-SHARE THRESHOLD IMPLEMENTATIONS FOR 4-BIT S-BOXES

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# Outline

- 1 Introduction
- 2 Threshold Implementation
- 3 Design
- 4 Experiments
- 5 Conclusion

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# Motivation, Contributions

## Motivation and Contributions

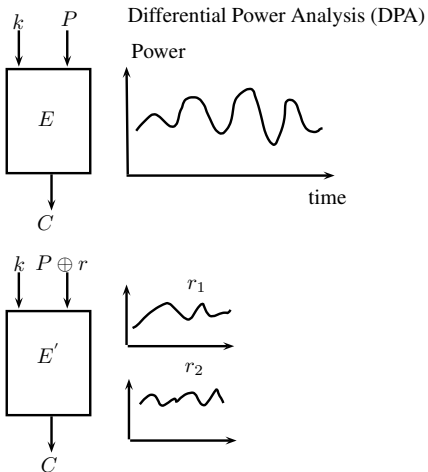
- 1 Reducing the hardware implementation of 3-share TI for a 4-bit S-box.
- 2 Implementation of improved 3-share TI of S-box of PRESENT.
- 3 Side Channel Attack experiments of improved approach.

So, what is **TI** or **3-share TI** and why do we need it?

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# Threshold Implementations I



2006, Nikova: Threshold Implementation Countermeasure.

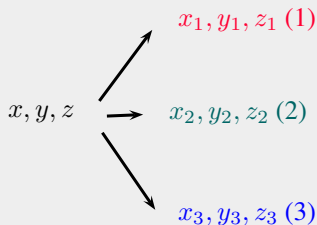
# Threshold Implementation II

## 3-share TI of function $f = z \oplus xy$

$$\mathbf{1} \quad x = x_1 \oplus x_2 \oplus x_3$$

$$y = y_1 \oplus y_2 \oplus y_3$$

$$z = z_1 \oplus z_2 \oplus z_3$$



# Threshold Implementation II

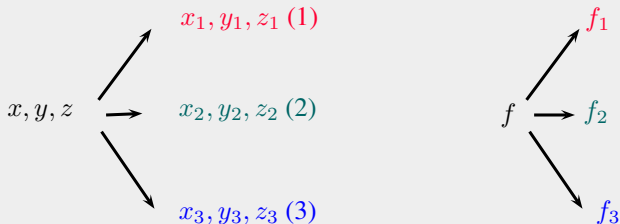
## 3-share TI of function $f = z \oplus xy$

$$1 \quad x = x_1 \oplus x_2 \oplus x_3$$

$$y = y_1 \oplus y_2 \oplus y_3$$

$$z = z_1 \oplus z_2 \oplus z_3$$

$$2 \quad f = f_1 \oplus f_2 \oplus f_3$$





# Threshold Implementation II

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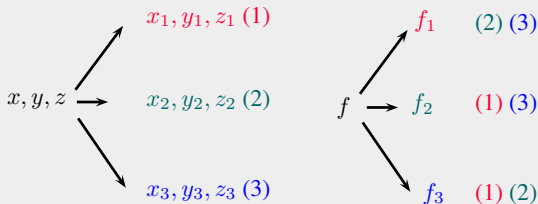
$$z = z_1 \oplus z_2 \oplus z_3$$

$$2 \quad f = f_1 \oplus f_2 \oplus f_3$$

$$3 \quad f_1 = z_2 \oplus x_2 y_2 \oplus x_2 y_3 \oplus x_3 y_2$$

$$4 \quad f_2 = z_3 \oplus x_3 y_3 \oplus x_1 y_3 \oplus x_3 y_1$$

$$5 \quad f_3 = z_1 \oplus x_1 y_1 \oplus x_1 y_2 \oplus x_2 y_1$$



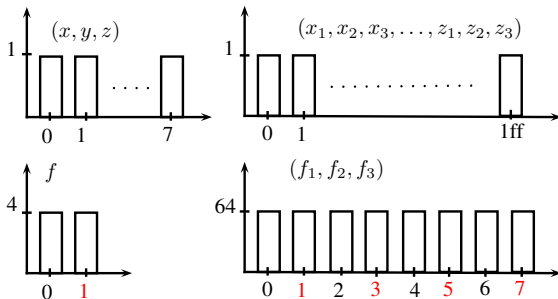
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## 3-share TI of function $f = z \oplus xy$

- 1  $x = x_1 \oplus x_2 \oplus x_3$   
 $y = y_1 \oplus y_2 \oplus y_3$   
 $z = z_1 \oplus z_2 \oplus z_3$
- 2  $f = f_1 \oplus f_2 \oplus f_3$
- 3  $f_1 = z_2 \oplus x_2 y_2 \oplus x_2 y_3 \oplus x_3 y_2$
- 4  $f_2 = z_3 \oplus x_3 y_3 \oplus x_1 y_3 \oplus x_3 y_1$
- 5  $f_3 = z_1 \oplus x_1 y_1 \oplus x_1 y_2 \oplus x_2 y_1$

$$(x, y, z) \longrightarrow (x_1, x_2, x_3, y_1, y_2, y_3, z_1, z_2, z_3)$$

$$f \longrightarrow (f_1, f_2, f_3)$$



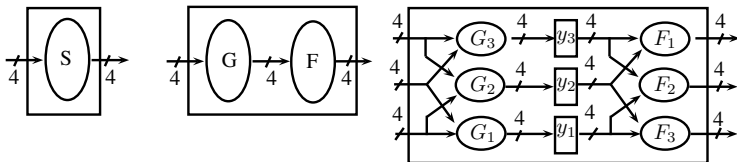
red:  $f_1 \oplus f_2 \oplus f_3 = f = 1$

black:  $f_1 \oplus f_2 \oplus f_3 = f = 0$

# 3-share TI to PRESENT S-BOX

Since degree of S-box  $S$  is 3  $\rightarrow$  4-share TI.

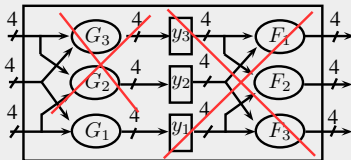
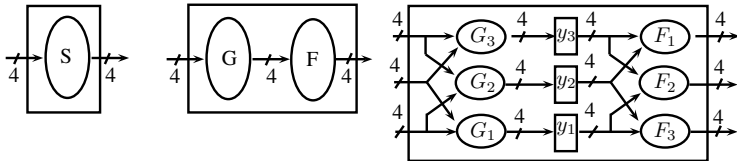
$S=F(G())$  where degrees of  $F$  and  $G$  are two.



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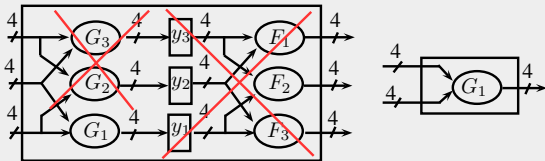
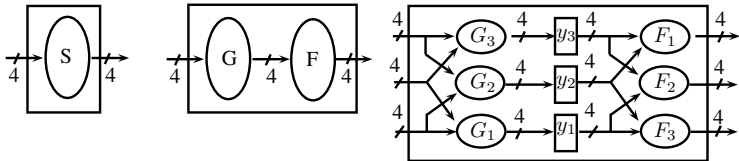
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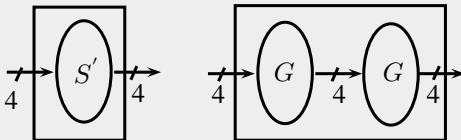
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# Reducing Hardware Implementation I

## 1st Observation

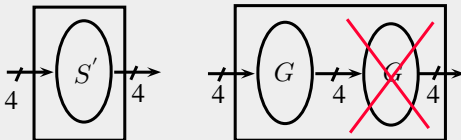
$$S' = G(G())$$



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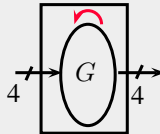
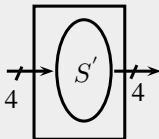
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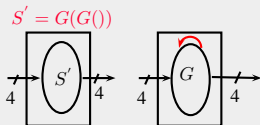
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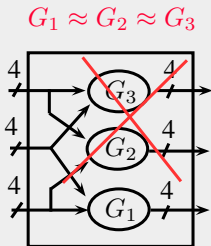


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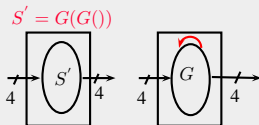


## 2nd Observation



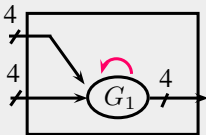
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## 2nd Observation

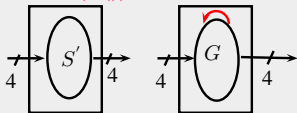
$$G_1 \approx G_2 \approx G_3$$



# Reducing Hardware Implementation I

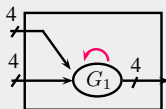
## 1st Observation

$$S' = G(G())$$



## 2nd Observation

$$G_1 \approx G_2 \approx G_3$$



## 3rd Observation



$$S'() = G(G())$$

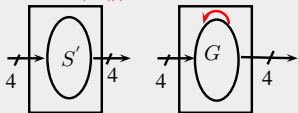
$$S(x) = A(S'(Bx \oplus c) \oplus d)$$

$$S(x) = A(G(G(Bx \oplus c)) \oplus d)$$

# Reducing Hardware Implementation I

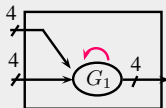
## 1st Observation

$$S' = G(G())$$



## 2nd Observation

$$G_1 \approx G_2 \approx G_3$$



## 3rd Observation

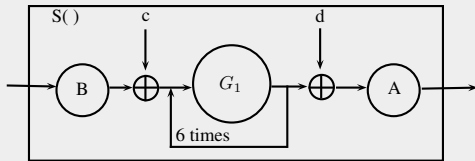


$$S'() = G(G())$$

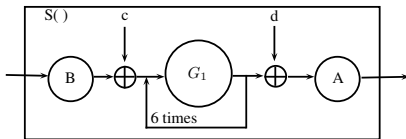
$$S(x) = A(S'(Bx \oplus c) \oplus d)$$

$$S(x) = A(G(G(Bx \oplus c)) \oplus d)$$

## Result



# Reducing Hardware Implementation II



## Improved 3-share TI to PRESENT S-BOX

1  $S := [12, 5, 6, 11, 9, 0, 10, 13, 3, 14, 15, 8, 4, 7, 1, 2]$

2  $G := [0, 4, 1, 5, 2, 15, 11, 6, 8, 12, 9, 13, 14, 3, 7, 10]$

3

$$A = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

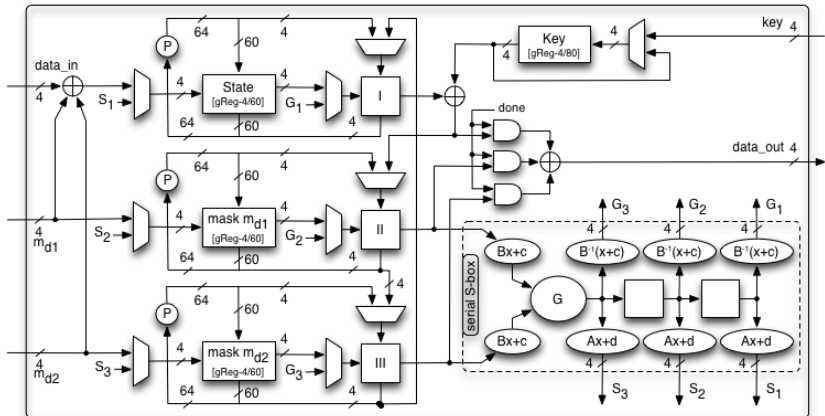
4  $c = (0001)_2 = 1, d = (0101)_2 = 5$

5  $S(x) = A(S'(Bx \oplus c) \oplus d) = A(G(G(Bx \oplus c)) \oplus d), \forall x \in \{0, \dots, 15\}$

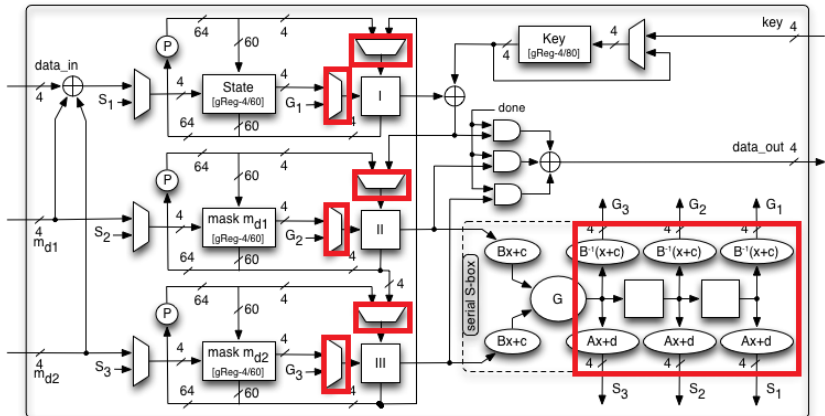
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## Improved Design I



## Improved Design I





# Improved Design II

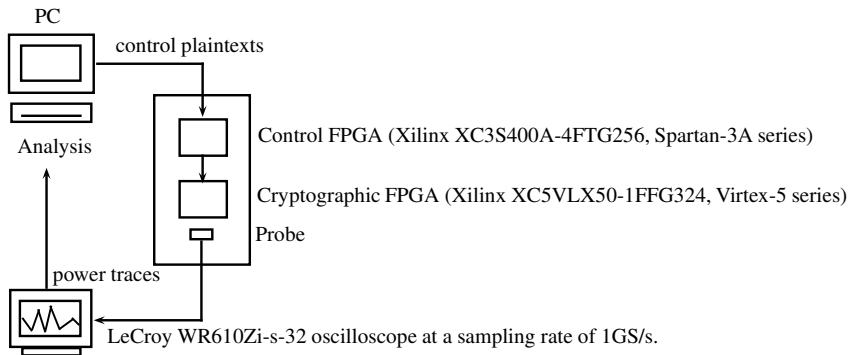
**Table :** Area savings for different implementation strategies.

Architecture Strategy	S-box Savings	Storage Strategy	S-box Area share	Overall Savings	Time Overhead
serial	-37.0%	D-FF + en s-FF + cg	11.4% 15.7%	-4.2% -5.8%	5.2
round-based	-40.6%	D-FF + en s-FF + cg	61.8% 67.9%	-25.1% -27.6%	3

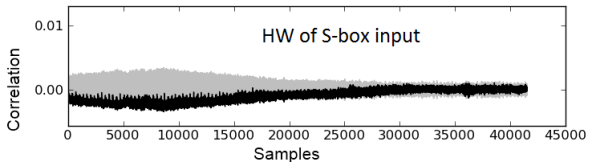
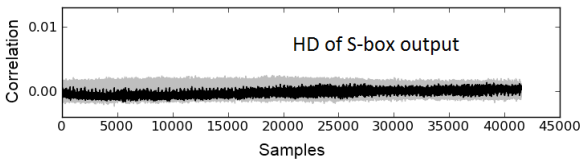
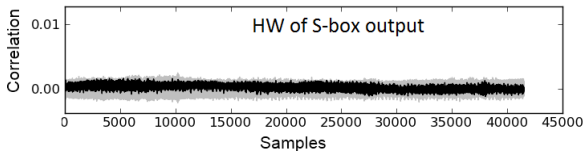
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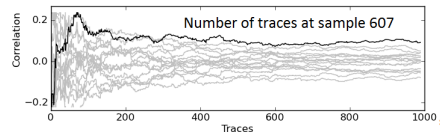
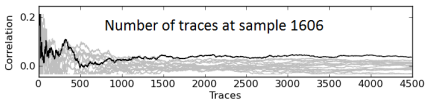
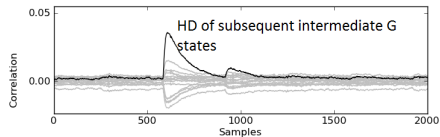
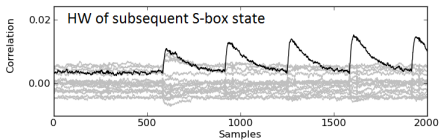
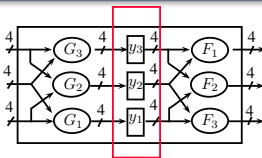
# Experiment setup



# The same security level: 5 million traces

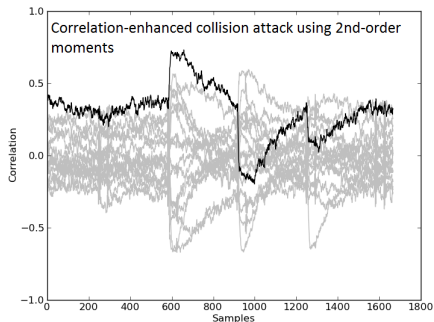
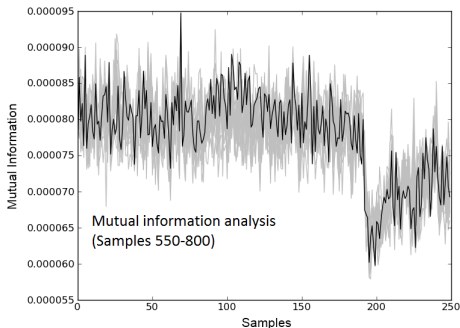


# Higher sensitive point for attack



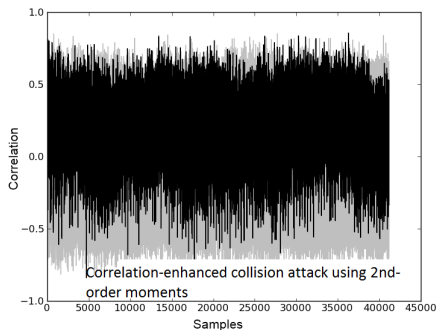
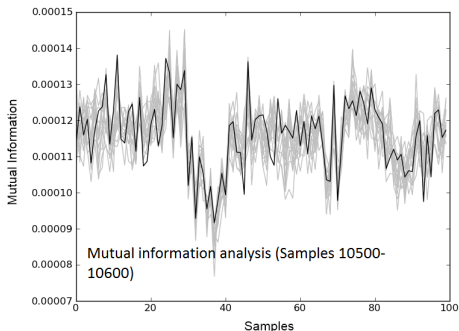
# MIA and correlation-enhanced collision attack I

- 1) 10,000,000 traces of this implementation and mounted both attacks targeting HD of consecutive outputs of the *G*-stage of original approach.
- 2) First successful practical MIA on TI.
- 3) Correlation-enhanced collision attack requires less traces than MIA.



# MIA and correlation-enhanced collision attack II

- 1) Sources for univariate leakage, e.g. the state update.
- 2) must be carefully serialized for every clock cycle, which is ongoing work.



# Wagner's zero-offset attack

- 1 Only works against the two-share masking scheme for the simulation.
- 2 Does not work against TI.
- 3 In order to attack against TI, the attack should be modified, i.e., by raising the mean-free measurement values to the power of three instead of squaring.
- 4 100 times worse than MIA (in simulation), which shows how sensitive this attack is against noise and why it does not work in practice.



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- 1 Two methodologies reducing the hardware implementation of TI are introduced.
- 2 A new design of 3-share TI for PRESENT's S-box is suggested.
- 3 Practical experiments show the security level of the new approach.

THANK YOU FOR LISTENING.

