Collision-Correlation Attack against some 1st-order Boolean Masking Schemes in the Context of Secure Devices

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Overview

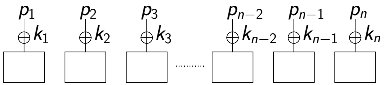
Linear Collision Attacks

Mechanisms of Collision Detection

2O-CPA on Mask-Reuse Scheme Implementation

Experiments and Results

Linear Collision Attack [Bog07, GS12]



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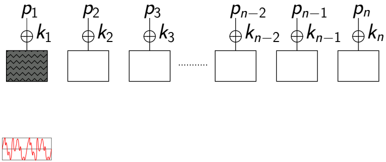

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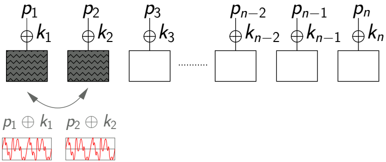

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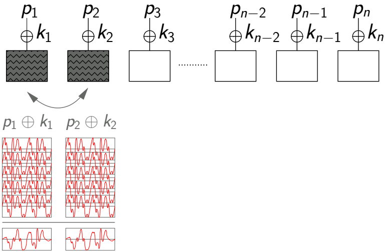

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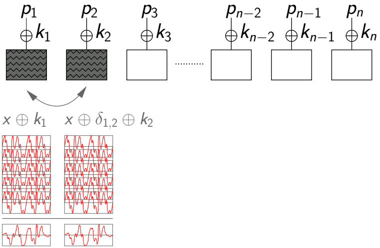

$$k_1 \oplus k_2 = \delta_{1,2}$$

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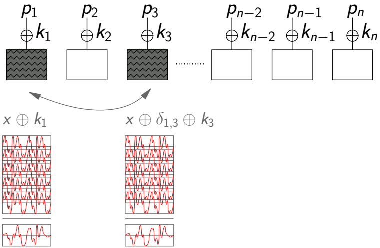

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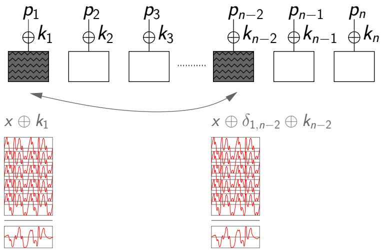

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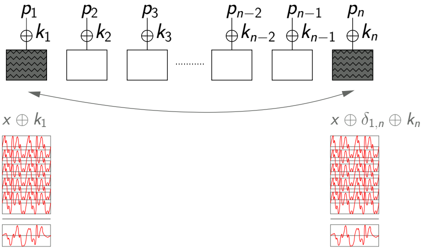

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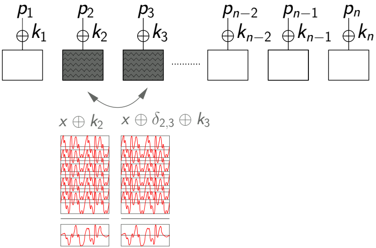

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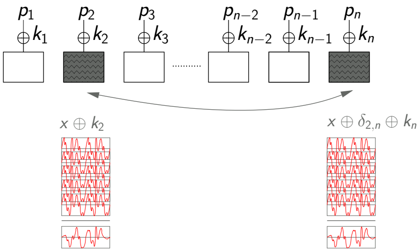

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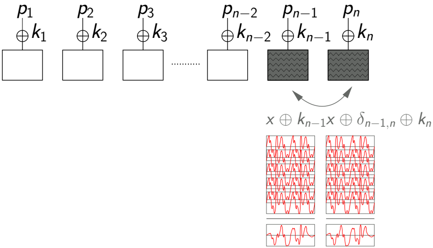

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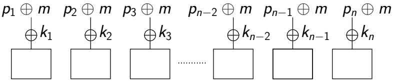
Linear Collision Attack on Masked Implementations [MME10, CFGRV11]



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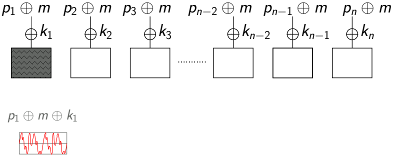

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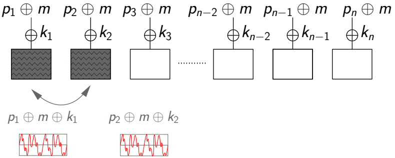

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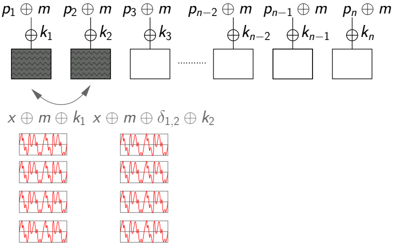

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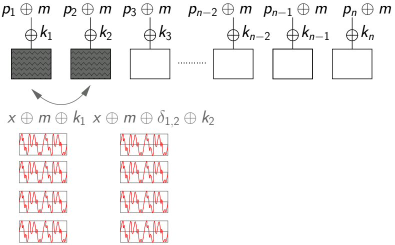

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Complexity Comparison After N Encryptions

- Unprotected Case

$$\Pr[\exists(i,j) \text{ s.t. } x_a^i \oplus x_b^j = k_a \oplus k_b] = \left(1 - (1 - \frac{1}{2^m})^{N^2}\right)$$

[AES] full rank system $\sim$ 7 messages [Bog07]

- Mask-Reuse Case

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Threshold Approach [Bog07, Bog08, CFGRV11] 1/2

Decide the presence (*resp.* absence) of collision from a set of traces pairs $((\ell_a^{i_k})_k, (\ell_b^{j_k})_k)$ such that $p_a^{i_k} \oplus p_b^{j_k} = \delta$.

- $\dfrac{1}{\bar{N}_\delta} \sum_x \mathrm{ED}(\bar{\ell}_{x,a}, \bar{\ell}_{x \oplus \delta, b}) < T$
- $\rho((\ell_a^{i_k})_k, (\ell_b^{i_k})_k) > T$

$\hookrightarrow$ Collision-based attack more efficient than CPA/2O-CPA

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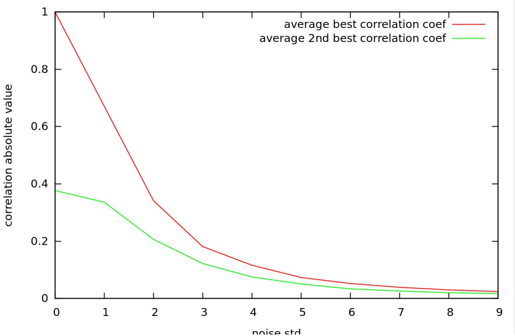

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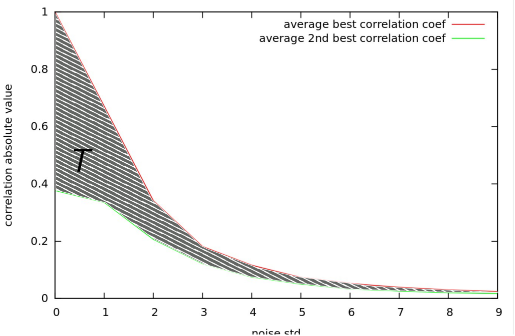

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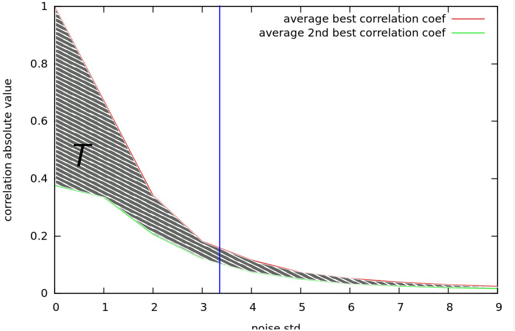
$S(p_2 \oplus k_2)$



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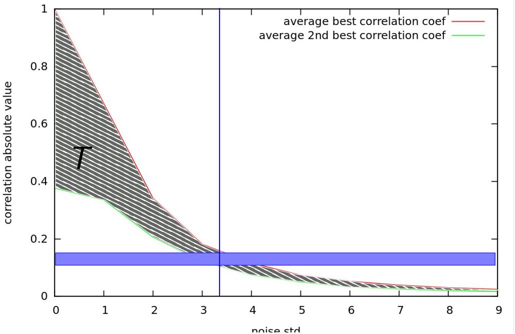

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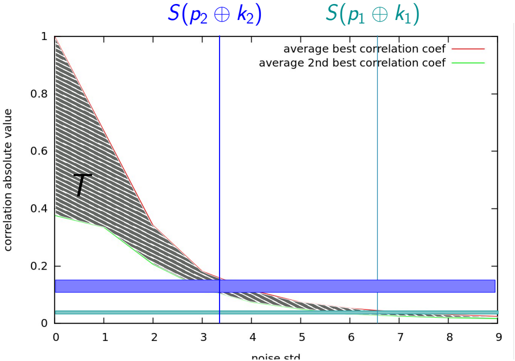

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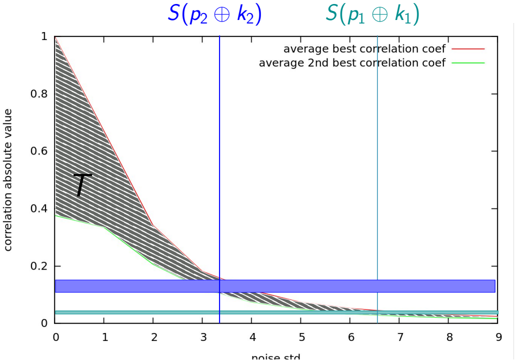

Limits of the approach

- T unknown and hard to guess when σ "high"
- different Ts for each pair of IVs

Unified Approach [GS12] 1/3

Decide the presence of collision from all the traces pairs $((\ell_a^{i_k})_k, (\ell_b^{j_k})_k)_\delta$ such that $p_a^{i_k} \oplus p_b^{j_k} = \delta$.

- $\min_\delta \frac{1}{\bar{N}_\delta} \sum_x \mathrm{ED}(\bar{\ell}_{x,a}, \bar{\ell}_{x \oplus \delta, b})$
- $\max_\delta \rho((\bar{\ell}_{x,a})_x, (\bar{\ell}_{x \oplus \delta, b})_x)$

$\hookrightarrow$ increase in message complexity

distinguishing values must be compared with something.

Unified Approach [GS12] 2/3

$$
\begin{aligned}
k_1 \oplus k_2 &= \delta_{1,2} \\
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k_{n-1} \oplus k_n &= \delta_{n-1,n}
\end{aligned}
$$

Unified Approach [GS12] 2/3

$$
\begin{array}{rcl}
k_1 \oplus k_2 & = & \delta_{1,2}, p_{1,2} \\
 & \vdots & \\
k_1 \oplus k_n & = & \delta_{1,n}, p_{1,n} \\
k_2 \oplus k_3 & = & \delta_{2,3}, p_{2,3} \\
 & \vdots & \\
k_{n-1} \oplus k_n & = & \delta_{n-1,n}, p_{n-1,n}
\end{array}
$$

Unified Approach [GS12] 2/3

$$
\begin{array}{lcll}
k_1 \oplus k_2 & = & 0, p_{1,2}(0); & \cdots ; \quad 2^m - 1, p_{1,2}(2^m - 1) \\
 & \vdots & & \\
k_1 \oplus k_n & = & 0, p_{1,n}(0); & \cdots ; \quad 2^m - 1, p_{1,n}(2^m - 1) \\
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\end{array}
$$

While $\underset{\delta_{1,2},\cdots,\delta_{n-1,n}}{\operatorname{argmax}} (p_{i,j}(\delta_{i,j}))$ is not a codeword

For $1 \leq a < b \leq n, \delta \in \mathrm{GF}(2^m)$

Do $p_{a,b}(\delta) \leftarrow p_{a,b}(\delta) \cdot \prod_{c \notin \{a,b\}} \sum_{\beta \in \mathrm{GF}(256)} p_{a,c}(\beta) \times p_{b,c}(\beta \oplus \delta)$

Unified Approach [GS12] 2/3

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\end{array}
$$

$\texttt{While} \underset{\delta_{1,2},\cdots,\delta_{n-1,n}}{\operatorname{argmax}} (p_{i,j}(\delta_{i,j}))$ is far from a codeword

$\quad \texttt{For } 1 \leq a < b \leq n, \delta \in \mathrm{GF}(2^m)$

$$\texttt{Do } p_{a,b}(\delta) \leftarrow p_{a,b}(\delta) \cdot \prod_{c \notin \{a,b\}} \sum_{\beta \in \mathrm{GF}(256)} p_{a,c}(\beta) \times p_{b,c}(\beta \oplus \delta)$$

Unified Approach [GS12] 3/3

$\hookrightarrow$ CPA (stochastic) more efficient than Collision-based attack
when the leakage function is not too far from the model basis

$\hookrightarrow$ extension to second order?
mask-reuse implementations

2nd-Order Correlation Power Analysis on Mask-Reuse Implementation

Classical Approach

$$\rho_{\hat{k}}\left((f_{\mathrm{opt}(\hat{z}_a^i)})_i, \mathcal{C}(\ell_m^i, \ell_{z_a^i \oplus m}^i)_i\right)$$

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Adapted Approach

$$\rho_{\hat{k}_a, \hat{k}_b}\left((f_{\mathrm{opt}(\hat{z}_a^i, \hat{z}_b^i)})_i, \mathcal{C}(\ell_{z_a^i \oplus m}^i, \ell_{z_b^i \oplus m}^i)_i\right)$$

2nd-Order Correlation Power Analysis on Mask-Reuse Implementation

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Adapted Approach

$$\rho_{\hat{k}_a, \hat{k}_b}\left(\left(f_{\mathrm{opt}(\hat{z}_a^i, \hat{z}_b^i)}\right)_i, \mathcal{C}\left(\ell^i_{z_a^i \oplus m}, \ell^i_{z_b^i \oplus m}\right)_i\right)$$

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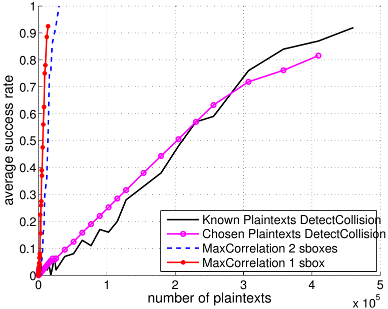
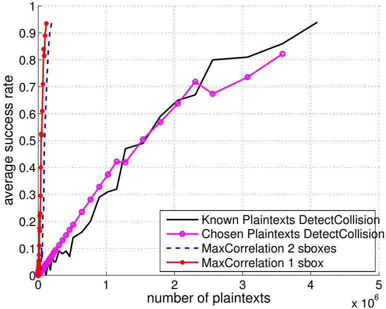
Simulations on two bytes

Simulations Hamming Weight + Gaussian Noise

$\sigma = 4$



$\sigma = 7$



Experiments on AES Mask-Reuse Implementation

Description

- component: 8-bit MCU
- side-channel: electromagnetic radiations
- sampling rate: 10G samples per seconds

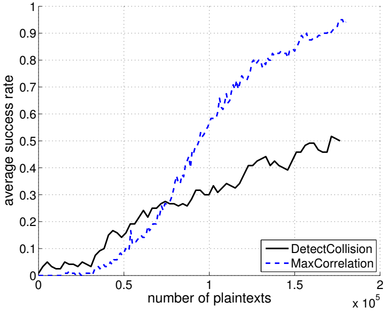
Noise Standard Deviation

	S-box 1	S-box 2	S-box 3	S-box 4
σ	6.0	3.7	3.4	3.3

Focusing on two bytes

Attacks on the MCU

Results on real traces



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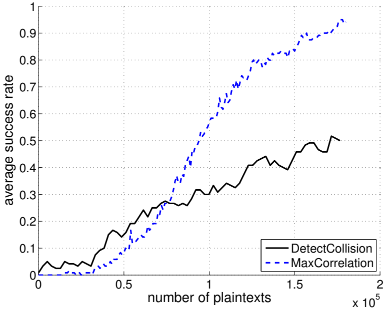
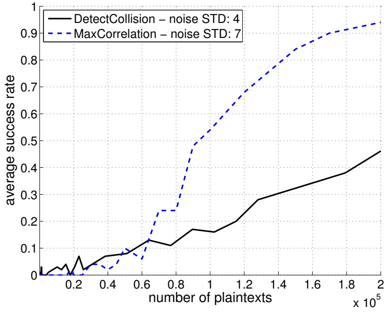


Simulations Co-co: $\sigma = 4$ Simulations 2O-CPA: $\sigma = 7$



Attacks on the whole 16-byte key

	Raw Attack	Hard Decoding	Soft Decoding
Co-co	> 200000	123000	50000
2O-CPA	160000	73000	X

To sum up (on our MCU)

- The use of decoding techniques helps a lot the attack efficiency.
- Collision-Correlation attack was less efficient that 2O-CPA (on our component).