# Algorithmic Approaches to Defeat Side Channel Analysis

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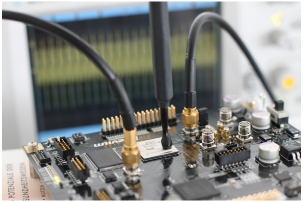
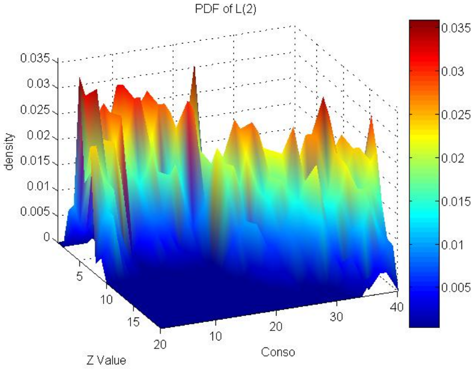
# Probability distribution function (pdf) of Electromagnetic Emanations

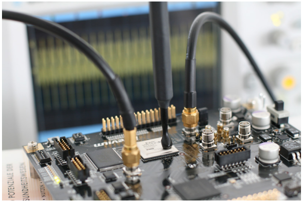
$$Z = S(X + k) \text{ with } X = 0 \text{ and } k = 2.$$

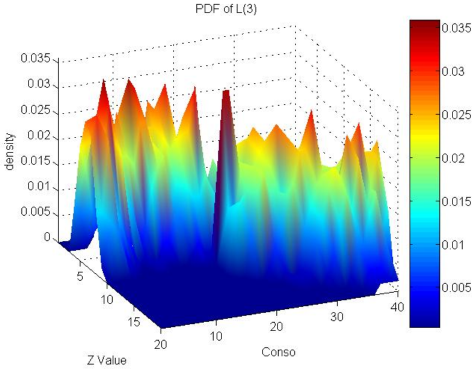




# Probability distribution function (pdf) of Electromagnetic Emanations

$$Z = S(X + k) \text{ with } X = 0 \text{ and } k = 3.$$





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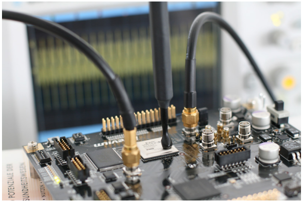

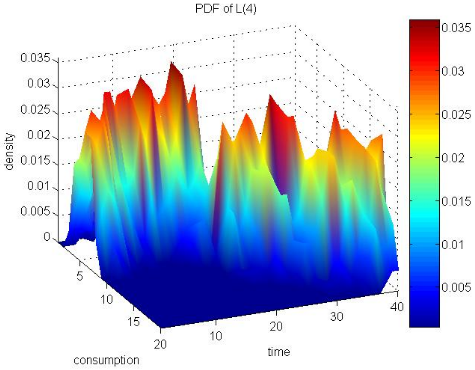

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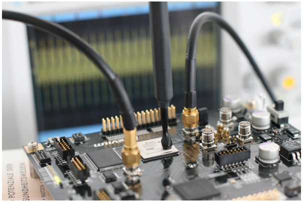

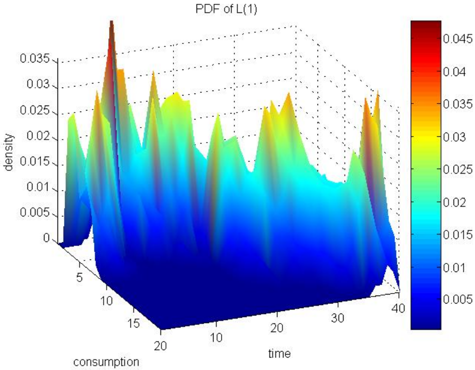

## Side Channel Attacks (SCA)

- Against **each** cryptosystem and **each** implementation, find the most efficient SCA.
  - Efficiency of an SCA?
  - Which attack parameters to improve?
  - SCA common trends?
  - Attacks *versus* Characterization!

## Countermeasures

- For **each** cryptosystem, find efficient/effective countermeasures.
  - Formally define the fact that a countermeasure thwarts an SCA?
  - Which countermeasure for which SCA?
  - What makes a cryptosystem more vulnerable to SCA than another?

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- Yes! Many *ad hoc* security analyses have been invalidated!
  - *e.g.* *GolicTymen02*, *AkkarBevanGoubin2004*, *FumaroliMayerDubois2007*, *CoronProuffRivain2007*, *ProuffMacEvoy2009*, *Debraize2012*, etc.

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- No! Practical Security $\neq$ Theoretical Security!
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  - or some physical phenomena are difficult to model (*e.g.* glitches)
  - or artefacts in acquisition chain behaviour *MoradiMische2013*

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## An attempt to sum-up

- proofs help **designers** to achieve measurable security
- do not prevent **evaluators** to test theoretically-impossible attacks

- Main Remark: SCA efficiency depends on the amount of noise in the observation.

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- Secret Sharing: randomly split $Z$ into $d$ shares $Z_1$, ..., $Z_d$:

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$$L_1 = \varphi(Z_1) + \mathcal{N}_1 \quad L_2 = \varphi(Z_2) + \mathcal{N}_2 \quad \cdots \quad L_d = \varphi(Z_d) + \mathcal{N}_d$$

  - all the $L_i$ are needed to get information on $Z$!
  - hence the adversary must combine all the $L_i$
  - lead to multiply the $\mathcal{N}_i$ altogether and to merge information and noise in a complex way.

## Adversary Game

In the implementation, find $d$ or less intermediate variables that jointly depend on a secret variable $Z$.

## Developer Game

Translate (Compile?) an implementation into a new one defeating the adversary.

Implementation = sequence of **elementary operations** which **read a memory location** and **write its result in another memory location**.

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Conclusion: need for another approach!

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  - Leakage : $L_i \sim x_i + \mathcal{N}(\mu, \sigma^2)$
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- Until now, two options exist to prove the security:
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  - the Information Bounded model.
- The two models have been recently unified in *DucDziembowskiFaust14*.

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- Recently *Belaid, Fouque and Barthe* developed automatic tools to generate security certificates.

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where $\psi$ is a security parameter depending on the noise.

- Security Proof goal: find a deterministic function $P$ s.t.:

$$\mathrm{MI}((X,k); (f_i(Z_i))_i) \leq P(1/\psi)$$

where $X$ is the plaintext and $k$ is the key.

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- Related to:
  - secret sharing *Shamir79*
  - design of error correcting codes with large dual distance *Massey93*

- Second Issue: how to securely process on shared data?
- Related to:
  - secure multi-party computation *NikovaRijmenSchläffer2008 ProuffRoche2011*
  - circuit processing in presence of leakage *GoldwasserRothblum2012*
  - efficient polynomial evaluation *CarletGoubinProuffQuisquater-Rivain2012,CoronProuffRoche2012*

- Linear Secret Sharing with parameters $n$ and $d$:
  - $n$ elements $Z_i$ such that

$$Z = \sum_i Z_i$$

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  - gives a general framework to describe and analyse all linear sharing schemes
  - links our problems with those of a rich community

- Linear Sharing = Encoding

$$\begin{pmatrix} Z & R_1 & \dots & R_{k-1} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 & 0 & \alpha_{1,k} & \dots & \alpha_{1,n} \\ 0 & 1 & 0 & 0 & \alpha_{2,k} & \dots & \alpha_{2,n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 & \alpha_{k,k} & \dots & \alpha_{k,n} \end{pmatrix} = \begin{pmatrix} Z & Z_1 & \dots & Z_{k-1} & Z_k & \dots & Z_n \end{pmatrix}$$

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$$
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- Actually masking order $= \min_{\vec{H} \in C^{\perp}} \mathrm{HW}(\vec{H}) - 1$ *Massey93*

- Boolean Sharing: encoding with the matrix

$$G = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & 1 \end{pmatrix}$$

implies $k = n - 1$.

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- Shamir's secret Sharing:
  - generate a random degree-$d$ polynomial $P(X)$ such that $P(0) = Z$
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- ... amounts to define a Reed-Solomon code with parameters $[n+1, d+1, \cdot]$ *McElieceSarwate81*.

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- ... amounts to define a Reed-Solomon code with parameters $[n+1, d+1, \cdot]$ *McElieceSarwate81*.
- Main issue: minimize $n$ for a given $d$.

- Multiplicative Masking *Gollic2002, GenelleProuffQuisquater2010*

$$Z \mapsto Z_0, \ldots, Z_d \text{ s.t. } Z_i \neq 0 \text{ and } Z = Z_0 \times \cdots \times Z_d$$

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- Affine Masking *vonWillich2001,FumarolliMartinelliProuffRivain2010*

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- Modular Additive Masking *Coron1999*

$$Z \mapsto Z_0, Z_1 \text{ s.t. } Z = Z_1 + Z_2 \mod \ldots$$

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- Affine Masking *vonWillich2001,FumarolliMartinelliProuffRivain2010*

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- Modular Additive Masking *Coron1999*

$$Z \mapsto Z_0, Z_1 \text{ s.t. } Z = Z_1 + Z_2 \mod \dots$$

- Homographic Masking *CourtoisGoubin2005*

$$Z \mapsto \frac{Z_0 \times Z + Z_1}{Z_2 \times Z + Z_3} \text{ or } \infty \text{ if } Z = -\frac{Z_3}{Z_2} \text{ or } \frac{Z_0}{Z_2} \text{ if } Z = \infty$$

- Leakage Squeezing

*MaghrebiGuilleyDanger2011,CarletDangerGuilleyMaghrebi2014*

$$Z \mapsto Z_0, Z_1 \text{ s.t. } Z = Z_0 \oplus Z_1 \text{ and } Z_i \in \mathcal{C}$$

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- Inner Product *BalaschFaustGierlichsVerbauwhede2012* and *BalaschFaustGierlichs2015*

$$Z \mapsto \mathbf{L}, \mathbf{R} \in \mathrm{GF}(2^n)^d \text{ s.t. } Z = \mathbf{L} \cdot \mathbf{R}$$

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- Much more difficult for non-linear functions (*i.e.* multiplication)

- Securing Multiplication *IshaiSahaiWagner2003*:
  - Input: $(a_i)_i$, $(b_i)_i$ s.t. $\bigoplus_i a_i = a$, $\bigoplus_i b_i = b$
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- Illustration of ISW scheme for $d = 2$:

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where the $r_{i,j}$ are a $((d+1)^2, d)$-sharing of $0$.

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- Actually, we can do it with $(d+1)^2/2$ random values instead of $(d+1)^2$.

## Securing any Polynomial evaluation

- Write the s-box $\mathrm{S} : \{0,1\}^n \to \{0,1\}^m$ as a polynomial function over $\mathrm{GF}(2^n)$:

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  - $(x+y) \longrightarrow (x_0+y_0), (x_1+y_1), \cdots, (x_d+y_d)$
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  *IshaiSahaiWagner2004*

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The masking complexity of $S$ is the minimal number of non-linear multiplications needed for its evaluation.

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For polynomials: amounts to find efficient decompositions;

- Knuth-Eve algorithm *VonZurGathenNoker2003*
- or the Cyclotomic Method *CarletGoubinProuffQuisquaterRivain2012*
- or Coron-Roy-Vivek's method *CoronRoyVivek2014*

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- So, use additive masking for affine transformations and multiplicative masking for power functions.

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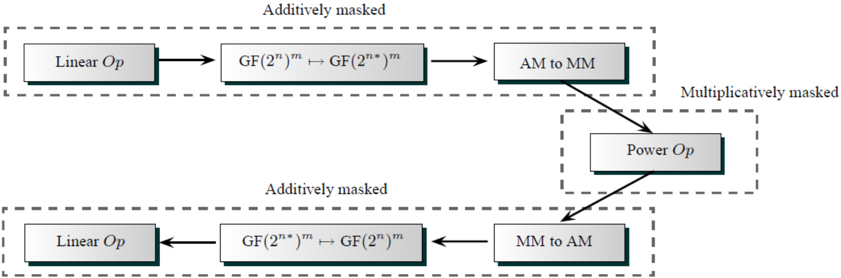
  - Soundness: for any power $e$, we have

$$(x \oplus \delta_0(x))^e = x^e \oplus \delta_0(x)$$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Recently extended to any order *at Asiacrypt2014*.

algebraic degree of a polynomial: greatest Hamming weight of the power of its monomials (with non-zero coefficients).

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Efficient (compared to SoA) for small $s$ or $n \ll d^s$.

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4. Find $t$ polynomials $p_i$ of algebraic degree $s$ and for $r+1$ linearized polynomials $\ell_i$ such that

$$S(x) = \sum_{i=1}^{t} p_i\big(q_i(x)\big) + \sum_{i=1}^{r} \ell_i\big(g_i(x)\big) + \ell_0(x) \ .$$

- The new method amounts to solve the linear system:

$$\begin{cases} \sum_{i=1}^{t} p_i\big(q_i(e_1)\big) + \sum_{i=1}^{r} \ell_i\big(g_i(e_1)\big) + \ell_0(e_1) & = & S(e_1) \\ \sum_{i=1}^{t} p_i\big(q_i(e_2)\big) + \sum_{i=1}^{r} \ell_i\big(g_i(e_2)\big) + \ell_0(e_2) & = & S(e_2) \\ \vdots & & \\ \sum_{i=1}^{t} p_i\big(q_i(e_{2^n})\big) + \sum_{i=1}^{r} \ell_i\big(g_i(e_{2^n})\big) + \ell_0(x) & = & S(e_{2^n}) \end{cases}$$

with (around) $t \times \frac{n^d}{s^d} + (r+1)n$ unknowns and $2^n$ equations.

- Necessary condition:

$$t \times \frac{n^d}{s^d} + (r+1)n \geqslant 2^n \quad .$$

- In practice, the lower bound was not achieved.

| | $n=4$ | $n=5$ | $n=6$ | $n=7$ | $n=8$ |
|---|---|---|---|---|---|
| $s=2$ (achieved) | **3** | **4** | **5** | **8** | **11** |
| $s=2$ (bound) | 2 | 4 | 5 | 6 | 9 |
| $s=3$ (achieved) | **2** | **3** | **3** | **4** | **4** |
| $s=3$ (bound) | 2 | 2 | 3 | 3 | 4 |

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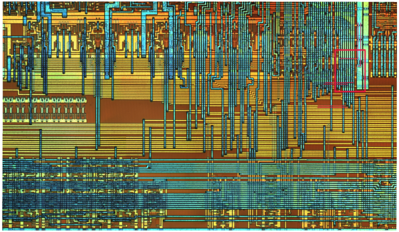

Thank you for your attention!
Questions/Remarks?